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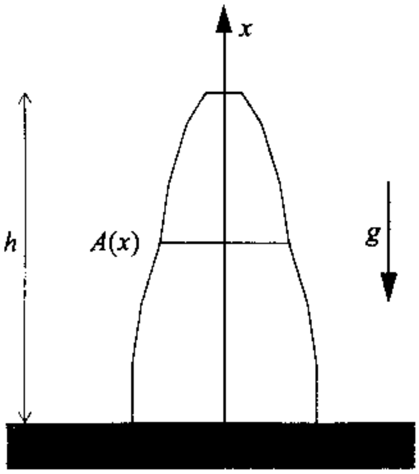
VASTE STOF MECHANICA (NAVSM)

Problem 1 Determine whether the following stress and strain fields are physically allowed:

$$[\sigma_{ij}] = \begin{bmatrix} ayz & dz^2 & ey^2 \\ dz^2 & bxz & fx^2 \\ ey^2 & fx^2 & cxy \end{bmatrix}, \quad [\varepsilon_{ij}] = \begin{bmatrix} x^2z & bxyz & 0 \\ bxyz^2 & az(x^2+y^2) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Here (x,y,z) are aliases for the Cartesian coordinates (x_i) $(i = 1\ldots3)$, while a through f are constants.

Problem 2 Chapter 2 discusses stresses and deformations in bodies subjected to external loading (tractions) on the boundaries. Bodies can also be loaded on interior points by so-called body forces. An example of this is gravity. It is not difficult to extend the equilibrium conditions to account also for gravity, but we will not do this here in general.



Instead, we here consider a purely one-dimensional problem, where a pile of height h is subject to gravity as the only external loading in the $-x$ direction (think, e.g., of a chimney at an electricity plant). The pile is assumed to be slender, i.e. cross-sectional dimensions are (much) smaller than h, so that variations in directions perpendicular to the x-axis can be neglected. Thus, geometry is defined by h and the cross-sectional area A as a function of x. When ρ denotes the mass density ([kg/m^3]), gravity (with gravitational acceleration g) exerts a force $\rho g A(x)dx$ on a slice of thickness dx. The average stress at the bottom of the pile is

$$\sigma(0) = -\frac{1}{A(0)}\int_0^h \rho g A(x)dx.$$

a. Derive the differential equation for equilibrium

b. Determine the shape, i.e. $A(x)$, such that the stress is constant throughout the pile. Find examples of structures that look like a pile and see if they possess this shape.

Problem 3 Von Mises (1883–1953) defined the following scalar representation of the stress tensor

$$\sigma_v = \sqrt{\frac{3}{2}\sigma'_{ij}\sigma'_{ij}},$$

where

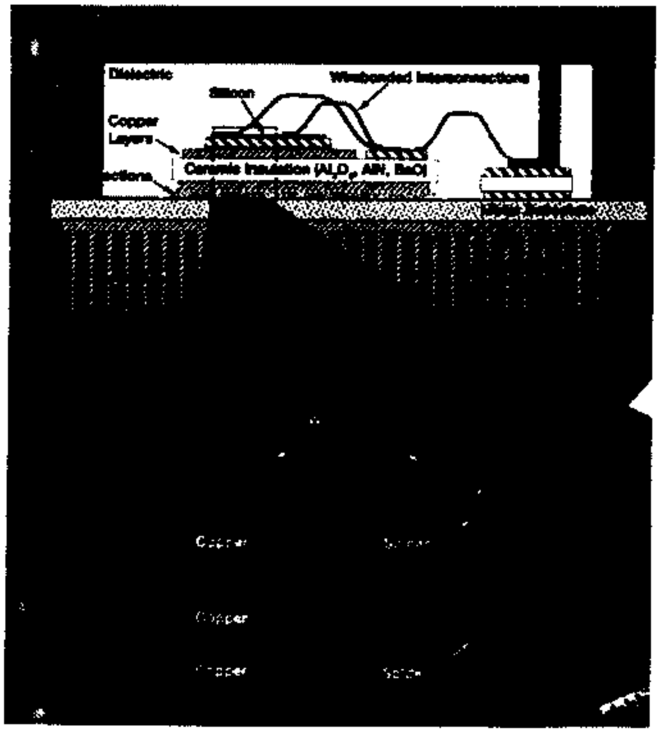
$$\sigma'_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}$$

is the stress deviator.

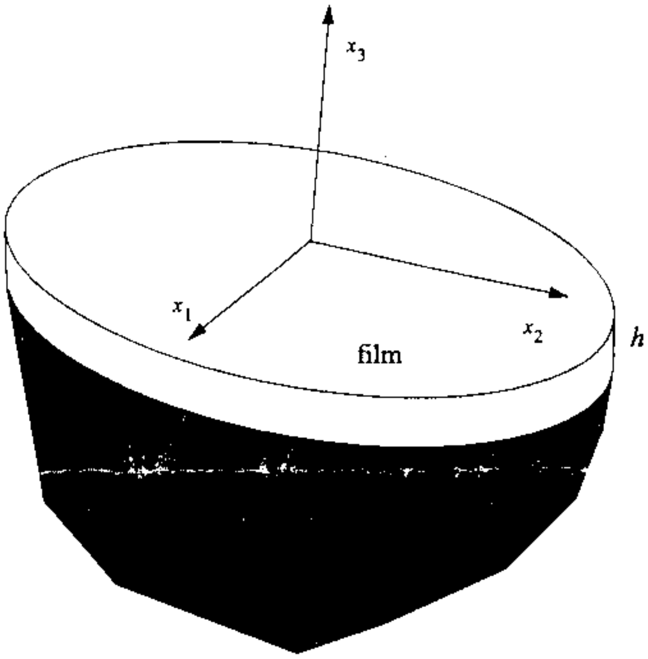
a. Express σ_v in terms of the stress σ in uniaxial tension.

b. Based on this result, propose a similar scalar-valued stress measure for shear.

Problem 4 Many applications involve thin films on relatively thick substrates. Examples include: wear and corrosion resistance coatings on steels in e.g. automotive applications, and metallic thin films on silicon wafers in electronics (chips are often built up from various layers, see figure). Even though the functional properties (i.e. electronic or optical) are most impor-



tant, the structural behaviour is also important in view of reliability. Structural behaviour is dictated by the stress that can develop in the film during operation. One of the most critical ones is thermal stress; this is stress that is caused by a difference in thermal expansion coefficient between film and substrate when the system is subjected to thermal cycles (e.g. switching on and off your computer). For example, the linear coefficient of thermal expansion of silicon (as the typical substrate material) is $\alpha_s = 4.2 \times 10^{-6}/C^\circ$ while that of aluminum for the film is $\alpha_f = 23.2 \times 10^{-6}/C^\circ$.



To get some feeling for the magnitude of these thermal stresses, the following simple problem is considered: a thin film (with thickness h) on an infinitely large substrate. Since the substrate is considered to be infinitely large, it will not deform as a consequence of stresses inside the film; the only thing it does is to expand thermally by

$$\varepsilon_{ij}^{\text{th}} = (\alpha_s - \alpha_f)\Delta T \delta_{ij}$$

where ΔT is the temperature difference from the stress-free state. Since the thermal expansion of the film is already incorporated in this way, the film itself will only deform elastically.

a. What is the stress state in the film, as a function of the coefficients of thermal expansion and the elastic properties of the film, assuming isotropic elasticity.

b. Calculate the maximal principal stress in the aluminum film on a silicon substrate after a temperature excursion of 600 C°.

c. Stress relaxation may occur in films due to plastic deformation. What is the preferred orientation of the slip systems in order that the stress gets relaxed most?